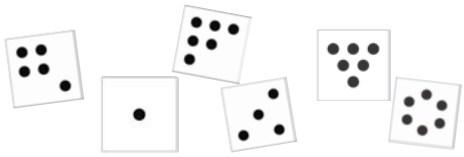

ABOUT THE MATH

If you watch and listen to how students interact with the games, you can learn a lot about what they know and what they're ready to learn. Once you see what they can do, you can help them take the next step. In this game, children practice:

- Reciting number words in the correct order
- Using one-to-one correspondence when counting
- Using the number name of the last object counted to represent the number of objects in the set (cardinality)
- Identifying the number of dots without counting (subitizing)

This section discusses some of the mathematical skills that children are building as they play. In mathematics, just as in their language and social/emotional skills, preschool children vary greatly in what they know and are able to do depending on their development and the experiences they've had. The more that mathematical ideas and play and talk becomes a regular part of their environment, the more they will learn.

Reciting number words in the correct order. Some children will know the number words but not be able to recite them in the correct order (i.e. 1, 2, 6, 4, 5, 10) or routinely skip a certain number (i.e. 1, 2, 3, 5, 6—always skipping 4). Recite the number list frequently in the day (when waiting for the bathroom, washing hands, passing out or cleaning up materials) and listen to children recite it, helping them where needed. Many children benefit from hearing rhythm to the list: "one, two, three; four, five, six;…" Tone can give salience too, particularly when children are stuck on a certain number. Saying: "Let's count together. One, two, three, four" each as if they're anticipating something special, and then "*five!*" at the end as if it were the special surprise we were all waiting for. Then: "See, *five* comes after four! Now you do it!"

The dot card games provide opportunity for children to practice the reciting the number list while counting the dots on the cards. As they play, you can pay attention to how different children are counting—what they are able to do and what they are not able to do yet.

One-to-one correspondence. When children have one-to-one correspondence that means they connect exactly one counting word to exactly one object. For example, a child counts three cubes as "one, two, three," touching each object only once and assigning only one counting word to each. This is a more complicated endeavor than it appears, because it requires two kinds of matches: (1) "matching a moment of time when the action occurs and a word is said"; and (2) "matching in space where the counting action points to an object once and only once" (NCTM, 2010, p. 13). Young children often recite the words and touch the objects at different rates, going through the right actions—reciting and touching—but not yet with the right meaning. You can model the correct counting to help them remember that each object needs one point and one number word; you don't skip any. Or you can say, "You might have missed one. Can you check?"

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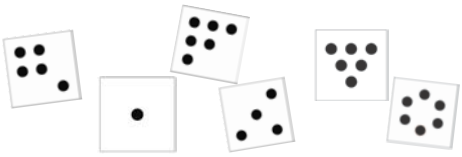
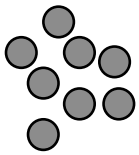


Sometimes children seem to think that a line of five blocks with big spaces between them has "more" than another line with the same number of blocks but more tightly spaced. They may be confusing the size of the configuration of objects and the total number of objects. You can try having them imagine that the objects are small toys or treats (or something else they'd want a lot of), and ask which collection they would want and why. Without negating what the child *has* done correctly—counting the number and assessing the visual "size"—this gives the child one extra experience thinking about quantity; experience, over time, will solidify the child's logic.

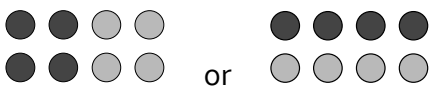
The dot games provide children the opportunity to practice one-to-one correspondence as the count the dots on the cards. The dots are arranged in different configurations—linear, rectangular, circular, and scattered—to give them practice with different formats.

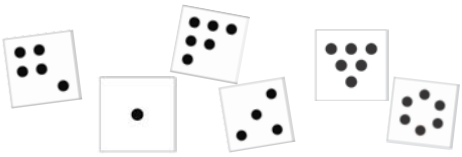
Cardinality. Children may count accurately (i.e., with one-to-one correspondence) but not yet recognize that the last number they said tells them the total amount in the collection. They have not yet achieved what is called 'cardinality.' After children count a set, we can ask them "How many do you have?" If they state the total number, they are showing an understanding of cardinality. If they recount the set, then they may not yet understand cardinality. Of course, they may also just have forgotten, or they may interpret the question to mean that they got the number wrong, and should check. They may even understand "How many?" not as a call to answer, but as a call to *act*, to *count*. We can help them by restating the total number after counting: "1, 2, 3. We have 3 pennies!"

Subitizing. Subitizing is *instantly seeing how many* for small quantities. For example, even preschoolers, for the most part, instantly recognize the number of fingers here  without needing to count "one, two" to know how many. But there are limits to what we see automatically. Even practiced adults rarely subitize quantities greater than 4 or 5. For example, this collection of dots is hard to subitize. We have to count, or mentally chunk it in parts, to know how many.



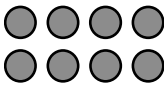
As children gain experience, they begin to see clusters they can subitize within a larger quantity (up to about 10) and can use those chunks to recognize the quantity immediately. For example, while 8 dots is generally too large to see immediately as 8, if they are arranged in a way that lets us easily perceive them as two groups of four, and if we 'know' that two 4s make 8, we "instantly" recognize the 8.





We *perceptually* subitize the two groups of four—two and four are both within perceptual limits—and then we *conceptually* subitize the 8.

Even without color to help us group the objects, a well-arranged collection can let our minds do that grouping for us, letting us quickly see how many.

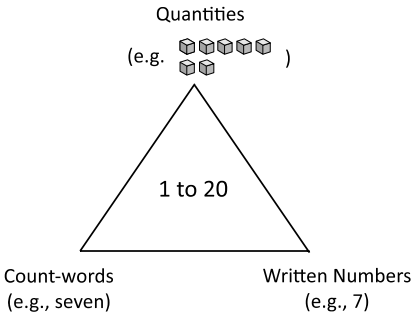


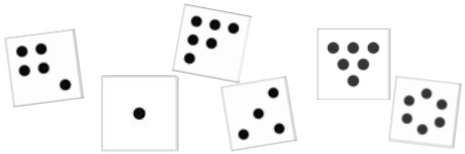
People can learn, in a limited way, learn to clump objects visually even when they are not arranged conveniently. For example, look back at that first mess of dots and see if you can see it in two groups you instantly know the size of. This is learnable, but more like a job skill (perhaps useful for quickly taking attendance) and not something worth spending children's time on.

You can incorporate practice with subitizing throughout the day. For example at snack time, "How many crackers do you have?" or at center time "How many dinosaurs/blocks/cars/dolls do you have?"

In the dot card games, encourage children to quickly 'see' how many dots are on the cards—particularly for cards with 3 or fewer dots. Also notice when they conceptual subitize and immediate 'know' 5 dots. You can ask them, "How did you see 5 dots so quickly? Did you group them?" They **do not** need to count one-by-one for every card. Sometimes children become less proficient at subitizing if they always count one-by-one. The cards have different configurations—linear, rectangular, circular, and scattered—to encourage children to see chunks of dots in different ways and encourage them to group the dots to subitize.

Written number symbols. Children can begin to connect *what* they are counting and *how* they are counting to written number symbols. They begin to understand how the quantity (number of blocks) relates to the counting words and to the written numerals. As children play the dot card games, you can help them bring these ideas together by pointing to a number line, number chart, or calendar in your classroom. Encourage children to see written numerals as symbols or tools to help them to express what they are thinking and to communicate easily with others (writing 5 is quicker than drawing 5 blocks).

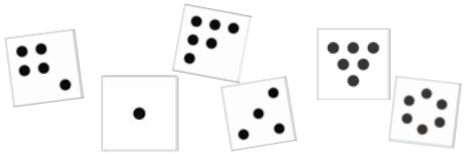




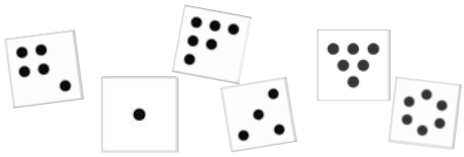
Progression of Number Concepts from 3- to 6-Years-Old (End of Kindergarten)

The lines between columns are intentionally fuzzy because the age is approximate. This progression is not to be used as an assessment or checklist, or to judge whether a child is ready to transition to Kindergarten. They represent expectations for children, but each child will reach these indicators at their own pace and their own way. These are meant to help you know what to expect; what learning may come first and what learning may come next for most children.

	@3 years old	@4 years old	@5 years old	@6 years old
Verbally count	Recites number names to 10 with occasional errors	Recites number words to 20 with occasional errors	Recites number words to 40 with occasional errors most likely in the teens	Counts to 100 by ones and tens
Count objects	Uses one-to-one correspondence for small groups of objects (under 5)	Uses one-to-one correspondence when counting (up to 10 objects)	Uses one-to-one correspondence when counting (up to 15)	Uses one-to-one correspondence when counting (up to 25)
Cardinality	Begins to understand that the last number tells the number of objects in a group	Understands the last number name said tells the number of objects counted up to 6 things.	Understands the last number name said tells the number of objects counted. Can count out n objects up to 10.	Counts to answer how many for up to 20 objects arranged in a line, array, circle, or up to 10 in a scattered configuration. Can count out n objects up to 20.
Subitizing	Begins to recognize the number of objects in a group of two or three without counting (subitizing)	Quickly sees how many for 1, 2, and 3 objects (subitize). May begin to subitize visually or conceptually up to 5 objects (by seeing 2 and 3).	Quickly sees how many with 1-10 objects when they are in a familiar arrangement; uses chunking for numbers 6-10 with a 5 group (array, fingers, dice pattern).	Quickly sees how many with 1-10 objects when they are in a familiar arrangement; uses chunking for numbers 6-10 with a 5 group (array, fingers, dice pattern).
Read and write numerals	Identifies numerals as being different than	Reads numerals 1-5	Reads numerals 1-10, begins to write some, such as 1, 3, 7	Reads and writes numerals 0-20



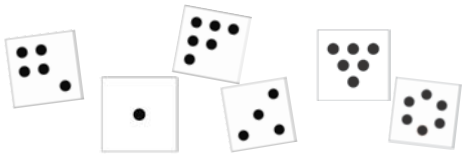
	letters and identifies some, such as 3.			
Compare numbers	Uses language to compare the number of objects in two groups (more, less, same)	Begins using strategies to find which is more for two numbers ≤ 5.	Uses counting to find which is more for two numbers ≤ 5. Uses the words less (fewer) than/more than/same as.	Identifies whether the number of objects in one group is greater than, less than, or equal another group of objects. Compares two written numerals between 1 and 10.
Counting on				Counts on from a given number instead of starting at 1 (e.g., starts at 3, counts 4, 5, 6)



OBSERVATIONS TO MAKE WHILE PLAYING

As you observe what your children are doing, support them to take the next step in their mathematical thinking by modeling, questioning, and explaining.

Do children		*Do children*
• Recite number words in the correct order?	OR	• Make mistakes in number word order after a certain number ("one, two, four") or skip certain numbers?
• Count dots one by one accurately?	OR	• Make mistakes in their counting such as skipping or double-counting dots?
• Arrive at how many quickly and efficiently (subitize)?	OR	• count one by one on some or all cards?
• Apply systematic methods to counting dot arrangements?	OR	• Count in random and unorganized fashion?
• Use visual patterns in dot configurations to count and combine subsets of dots to find a total count?	OR	• Apply the same counting strategy on all cards even when the dot configurations on some cards allow for more efficient counting methods?



BOOK LINKS

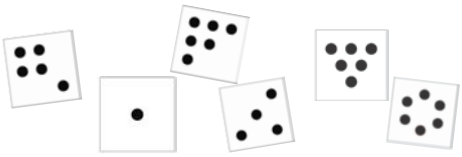
Ten Black Dots *by Donald Crews*
Classic counting picture book: one black dot makes a sun, 2 black dots the eyes of a fox, and three black dots a snowman, etc. As you read, have the group count the dots on the page together. This book is a great compliment to the dot card games where kids are practicing one-to-one counting and subitizing. As an extension have kids put 1-10 circle stickers on a page and draw their own designs. You can use this book to extend the idea that numbers are composed of parts by finding the smaller numbers of dots that are part of the whole. For example, on the 6 dot page one hand holds 3 new marbles while the other hand holds 3 old marbles—3 and 3 are 6. The train has 4 pairs of 2 wheels that make 8 wheels (dots) total.

Fish Eyes: A Book You Can Count On by Lois Elhert
This counting book is a pleasure to read aloud with beautiful, vivid illustrations. The narrator imagines she has turned into fish and to "flip down rivers and splash in the sea." One each page, children can count the fish 1 to 10. The little narrator fish includes a simple addition problem on each page such as, "4 striped fish plus me makes 5." Children enjoy counting the fish or sometimes the fish eyes as you read the book. For a challenge, they can try the "plus one" problem on each page. At the art table, children would enjoy making their own illustration of fish to count. To make their fish, they could glue color bits of paper on to their page or use crayons, markers, or paint. Children enjoy narrating their own picture and having you write down what they say.

Five Little Monkeys *by Eileen Christelow*
This fun book and song can be used to teach pattern, counting backwards, knowing one less, and understanding cardinality. For pattern, you can draw kids attention to the lines of the song that repeat each time and see if they know what changes—there is one less monkey jumping (rather than a growing pattern, it's a shrinking pattern). You can ask children to act out the book by having five children pretend to be the monkeys and someone pretend to be the mama. They can see how the number of children (monkeys) jumping decreases by one each time the pattern (verse of the song) repeats. To practice the pairs that make 5, you can ask children how many monkeys *are not* jumping (if 3 are jumping, 2 are not jumping).

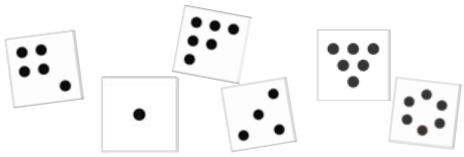
The Very Hungry Caterpillar by Eric Carle
This popular book is great for talking to children about 'how many in all' (cardinality) and comparing numbers. After you have read the book to the whole group, have book discussions in small groups. Ask the children questions such as "How many apples did he eat through? Pears? Plums? Strawberries?" See if they can name the total number rather than recount each time. Also notice if they are subitizing or counting one by one. Give the children practice comparing numbers by holding the book open to the pages that are cut out for each fruit. Ask children questions such as, "How many more pears did he eat than apples? How many more oranges than strawberries? How many more oranges than apples?"

How Many Snails?: A counting book by Paul Giganti, Jr., illustrated by Donald Crews



Walking to the meadow, lake, library, park, bakery, toy store, and other stops, the author wonders 'how many?' about a variety of different objects and in different combinations; such as: "How many snails were there? How many snails had striped shells? How many snails had striped shells and stuck their heads out?" This is a fun counting book to use as a read aloud and then for children to browse on their own counting all the objects and sorting them into different groups.

Quack and Count *by Keith Baker*
The seven ducklings in the books split into all the whole number combinations that make seven. First children count all seven ducks, then the ducks slide, hide, chase, splash, and quack in the combinations 6 + 1; 5 + 2; 4 + 3; 3 + 4; 2 + 5; 1 + 6, and finally all seven fly. This book helps children understand that numbers are composed of smaller numbers. This is the same mathematical idea in the *How Many Are Hiding* game—numbers are composed of parts that make up the whole.



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