ABOUT THE MATH:
HOW MANY ARE HIDING?

If you watch and listen to how students interact with the games, you can learn a lot about what they know and what they are ready to learn. Once you see what they can do, you can help them take the next step in their learning. In this game, children practice:

- Reciting number words in the correct order
- Using one-to-one correspondence when counting
- Using the number name of the last object counted to represent the number of objects in the set (cardinality)
- Identifying the number of dots without counting (subitizing)
- Comparing numbers
- Composing and decomposing numbers
- Working memory: creating a mental image of the initial string and notes what is missing after some are covered

This section discusses some of the mathematical skills that children are building as they play. Preschool children will vary in what they know and are able to do in mathematics just as in their language and social/emotional skills. The more practice and exposure they have to mathematics they more they will learn.

In this game, children practice reciting number words in the correct order, using one-to-one correspondence when counting, counting to know how many (cardinality), subitizing, comparing numbers, and composing and decomposing numbers. They also practice several executive function skills: keeping track (working memory), and self-regulation. In the beginning version of this game, children close their eyes while the teacher places tokens or other objects on the table. Children open their eyes and say how many they see. Teachers can vary the number of objects and their configuration (linear, array, scattered, etc). Next, children have to put out 3 to 5 tokens (or other objects). Then children close their eyes and the teacher covers some, all, or none of the tokens. Children open their eyes and say how many they see, then how many are hiding (how many they can't see). As children gain more practice, teachers can increase the number of objects and speed up the came so children are just naming how many are hiding. Children also enjoy doing the covering—in this case they still need to do the work to make sure their partner gets it right!

Reciting number words in the correct order. Some children will know the number words but not be able to recite them in the correct order (i.e. 1, 2, 6, 4, 5, 10) or routinely skip a certain number (i.e. 1, 2, 3, 5, 6—always skipping 4). Help them to learn the correct order by saying the number list with them and helping them where they are getting stuck; "Five comes after four, like this: one, two, three, four, *five*! Now *you* say it." Recite the number list frequently in the day (when waiting for the bathroom, passing out or cleaning up materials) and listen to

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children recite it, helping them where needed. Many children benefit from hearing rhythm to the list: "one, two, three; four, five, six;…" Tone can give salience, too, saying "one, two, three, four" each as if they're anticipating something special, and then "*five!*" at the end as if it were the special surprise we were all waiting for.

The *How Many Are Hiding* game provides opportunity for children to practice the reciting the number list to count the tokens. As they play, you can pay attention to how different children are counting—what they are able to do and what they are not able to do yet.

One-to-one correspondence. When children have one-to-one correspondence that means they recite counting words in sequence. They connect exactly one counting word to exactly one object. For example, a child counts three cubes as "one, two, three," touching each object only once and assigning only one counting word to each. This is a more complicated endeavor than it appears, because it requires two kinds of matches: (1) "matching a moment of time when the action occurs and a word is said"; and (2) "matching in space where the counting action points to an object once and only once" (NCTM, 2010, p. 13). Young children often recite the words and touch the objects at different rates, going through the right actions—reciting and touching—but not yet with the right meaning. You can model the correct counting to help them remember that each object needs one point and one number word; you don't skip any. Or you can say, "You might have missed one. Can you check?"

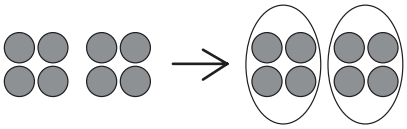
Some children may think that a line of five blocks with big spaces between tokens has more than another line of five blocks placed closely together even though they have the same number. They may be confusing the size of the configuration of objects and the total number of objects. You can try having them imagine that the objects are something that they want a lot of (such as matchbox cars or M&Ms), and ask which pile they would want and why. Without negating what the child *has* done correctly—counting the number and assessing the visual "size"—this gives the child one extra experience thinking about quantity; experience, over time, will solidify the child's logic.

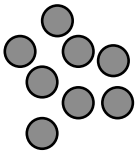
This game provide children the opportunity to practice one-to-one correspondence as they count the tokens.

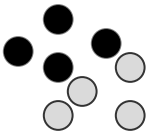
Cardinality. Before cardinality, children may count accurately (i.e., with one-to-one correspondence) but not yet recognize that the last number said when counting tells them the total amount in the collection. After children count a set, we can ask them "How many do you have altogether?" If they state the total number, they are showing an understanding of cardinality. If they recount the set, then they may not understand cardinality yet. We can help them by restating the total number after counting: "1, 2, 3. There are 3 tokens in all."

An aspect of cardinality is counting out objects to a certain number. When playing How *Many Are Hiding*, ask the children to 'set up' the game by counting out the number of tokens you need to play. Children have to hold the number of tokens they are counting out in their head, count correctly, and know to stop when they reach that number. This is another way to practice this skill.

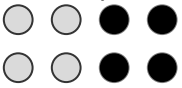
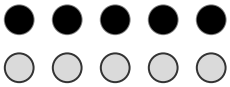

Subitizing. An essential aspect to building students' foundational understanding of cardinality is "subitizing." Subitizing is *instantly seeing how many* for small quantities. For example, instantly recognizing that two fingers,  , is two. Most children (and adults) do not need to count "one, two" to know how many. Even practiced adults generally do not subitize quantities greater than 4 or 5 but, as children gain experience, they will begin to see clusters of 1, 2, and 3 within a larger quantity (up to about 10) and then be able to use those chunks to recognize the quantity immediately. For example, while 8 dots is generally too large to see immediately as 8, we can mentally "chunk" them into two groups of four and 'know' 8 very quickly.



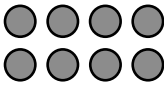
This process is called conceptual subitizing. Even for adults, this collection of dots is hard to subitize.  We have to count to know how many. But if they're somehow clumped, we recognize more quickly how many there are in the small parts, and if we *know* that two 4s make 8, then we can "instantly see" how many.



And if they're arranged, it's even easier.

  or

Even without color to help us group the objects, a well-arranged collection can let our minds do that grouping for us, letting us quickly see how many.



You can incorporate practice with subitizing throughout the day. For example at snack time, "How many crackers do you have?" or at center time "How many dinosaurs/blocks/cars/dolls do you have?"

The game *How Many Are Hiding*, encourages children to quickly 'see' how many tokens there are—especially when you play with 5 or fewer. Notice when they conceptually subitize and immediate 'know' 5 dots. You can ask them, "How did you see 5 dots so quickly? Did you group them?" They **do not** need to count one-by-one every time. Sometimes children become less proficient at subitizing if they always count one-by-one.

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Comparing Numbers. Children must have experiences that help them develop a 'mental number line'—a spatial representation of quantity that helps the reason about which is more and how much more. Children can reason that 7 is more than 5 because 7 comes later in the counting sequence and on the number line. Later, they can determine how many more by knowing that 5 is contained in 7 (a part of 7) and the "other part", the leftover, is 2. Some studies have found that lower income children can be several years behind their middle-income peers in being able to compare numbers and in their development of a mental number line (Griffin, Case, & Siegler, 1994). Most people tend to exaggerate the distances between smaller, more familiar numbers—1, 2, 3, 4—and underestimate differences between numbers at the higher end. Think about how "far apart" one hundred and one thousand are. Are one thousand and one million similarly spaced? One million and one billion? Helping children gain an understanding of the number line and estimating relationships between numbers on the number line are important for later mathematical understanding.

The game *How Many Are Hiding,* has children practice their counting and subitizing. It also helps children see that bigger numbers are composed of smaller numbers and thus helps to build a sense of number magnitude—that each counting number is just one more than the previous number.

Composing and Decomposing Numbers (i.e. recognizing that numbers (or sets of objects) can be combined or separated to make another number). Through everyday experiences, toddlers learn that a whole is made up of smaller parts and that the parts are smaller than the whole. (There can be 4 paintbrushes and if 3 of them are in use, only 1 paintbrush is left). Children also learn early on that objects can be combined in different orders to produce the same result—you can put the 3 blue trucks or the 3 red trucks in the bucket first but you still get 6 trucks (commutative property). By 4 or 5 years of age children can recognize that smaller numbers are 'hiding inside' larger numbers such as 2 and 3 are hiding inside 5 (as are 4 and 1).

The game *How Many Are Hiding,* explicitly has children practice these skills by counting how many in the whole (original set), covering some objects, stating how many children can see (first part), then stating how many children can't see (second part). For 3 and 4 year olds, just stating what they *can* see after some are covered may be enough. For 4 and 5 years old they may be ready to skip the first two steps and just state what is hiding. The oldest children may be ready for higher numbers, up to 10.

Working Memory. This game requires children to hold information in mind and use it to answer the question: *How many are hiding*? Children create a mental image of the initial string and note what is missing after some are covered. Working memory is strongly tied to school success in all domains—mathematics, reading, writing, etc.

Progression of Number Concepts from 3- to 6-Years-Old (End of Kindergarten)

The lines between columns are intentionally fuzzy because the age is approximate. This progression is not to be used as an assessment or checklist, or to judge whether a child is ready to transition to Kindergarten. They represent expectations for children, but each child will reach these indicators at their own pace and their own way. These are meant to help you know what to expect; what learning may come first and what learning may come next for most children.

	@3 years old	@4 years old	@5 years old	@6 years old
Verbally count	Recites number names to 10 with occasional errors	Recites number words to 20 with occasional errors	Recites number words to 40 with occasional errors most likely in the teens	Counts to 100 by ones and tens
Count objects	Uses one-to-one correspondence for small groups of objects (under 5)	Uses one-to-one correspondence when counting (up to 10 objects)	Uses one-to-one correspondence when counting (up to 15)	Uses one-to-one correspondence when counting (up to 25)
Cardinality	Begins to understand that the last number tells the number of objects in a group	Understands the last number name said tells the number of objects counted up to 6 things.	Understands the last number name said tells the number of objects counted. Can count out n objects up to 10.	Counts to answer how many for up to 20 objects arranged in a line, array, circle, or up to 10 in a scattered configuration. Can count out n objects up to 20.
Subitizing	Begins to recognize the number of objects in a group of two or three without counting (subitizing)	Quickly sees how many for 1, 2, and 3 objects (subitize). May begin to subitize visually or conceptually up to 5 objects (by seeing 2 and 3).	Quickly sees how many with 1-10 objects when they are in a familiar arrangement; uses chunking for numbers 6-10 with a 5 group (array, fingers, dice pattern).	Quickly sees how many with 1-10 objects when they are in a familiar arrangement; uses chunking for numbers 6-10 with a 5 group (array, fingers, dice pattern).
Compare numbers	Uses language to compare the number of objects in	Begins using strategies to find which is more for two numbers ≤ 5.	Uses counting to find which is more for two numbers ≤ 5. Uses the	Identifies whether the number of objects in one group is greater than, less

	two groups (more, less, same)		words less (fewer) than/more than/same as.	than, or equal another group of objects. Compares two written numerals between 1 and 10.
Composing and Decompos-ing Numbers	Knows the whole is bigger than the parts, but may not correctly quantify.	Beginning to know number combinations up to 4 or 5 (3 has 2 and 1 in it).	Uses objects or fingers to decompose small numbers (3, 4, 5) into its parts (5 has 4 and 1 and 3 and 2 inside it). Names parts of numbers up to 5.	Decomposes numbers to 10 into pairs using objects, drawings, and/or equations. Knows the pairs that make 10. Fluently adds and subtracts within 5.

OBSERVATIONS TO MAKE WHILE PLAYING

As you observe what your children are doing, support them to take the next step in their mathematical thinking by modeling, questioning, and explaining.

Do children		*Do children*
• Recite number words in the correct order?	OR	• Make mistakes in number word order after a certain number ("one, two, four") or skip certain numbers?
• Count tokens one by one accurately?	OR	• Make mistakes in their counting such as skipping or double-counting dots?
• Arrive at how many quickly and efficiently (subitize)?	OR	• Count one by one on some or all cards?
• Apply systematic methods to counting?	OR	• Count in random and unorganized fashion?
• Know if a number is more, less, or equal to another number?	OR	• Make mistakes comparing numbers?
• Accurately count out the right number of tokens?		• Make mistakes counting out the tokens or are uncertain how to?
• Accurately say how many are hiding for a given number (5 or less)?		• Not know how many are hiding for a given number?

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BOOK LINKS

Ten Black Dots *by Donald Crews*
Classic counting picture book: one black dot makes a sun, 2 black dots the eyes of a fox, and three black dots a snowman, etc. As you read, have the group count the dots on the page together. This book is a great compliment to the dot card games where kids are practicing one-to-one counting and subitizing. As an extension have kids put 1-10 circle stickers on a page and draw their own designs.

You can use this book to extend the idea that numbers are composed of parts by finding the smaller numbers of dots that are part of the whole. For example, on the 6 dot page one hand holds 3 new marbles while the other hand holds 3 old marbles—3 and 3 are 6. The train has 4 pairs of 2 wheels that make 8 wheels (dots) total.

Quack and Count *by Keith Baker*
The seven ducklings in the books split into all the whole number combinations that make seven. First children count all seven ducks, then the ducks slide, hide, chase, splash, and quack in the combinations 6 + 1; 5 + 2; 4 + 3; 3 + 4; 2 + 5; 1 + 6, and finally all seven fly. This book helps children understand that numbers are composed of smaller numbers. This is the same mathematical idea in the *How Many Are Hiding* game—numbers are composed of parts that make up the whole.

Fish Eyes: A Book You Can Count On by Lois Elhert
This counting book is a pleasure to read aloud with beautiful, vivid illustrations. The narrator imagines she has turned into fish and to "flip down rivers and splash in the sea." One each page, children can count the fish 1 to 10. The little narrator fish includes a simple addition problem on each page such as, "4 striped fish plus me makes 5." Children enjoy counting the fish or sometimes the fish eyes as you read the book. For a challenge, they can try the "plus one" problem on each page. At the art table, children would enjoy making their own illustration of fish to count. To make their fish, they could glue color bits of paper on to their page or use crayons, markers, or paint. Children enjoy narrating their own picture and having you write down what they say.

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