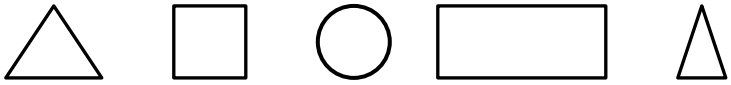
ABOUT THE MATH

If you watch and listen to how students interact with the games, you can learn a lot about what they know and what they're ready to learn. Once you see what they can do, you can help them take the next step. In these card games, children practice:

- Naming shapes
- Recognizing shape attributes
- Recognizing numerals
- Shifting rules, keeping track (working memory), regulating themselves during game play (executive functions)

This section discusses some of the mathematical skills that children are building as they play. In mathematics, just as in their language and social/emotional skills, preschool children vary greatly in what they know and are able to do depending on their development and the experiences they've had. The more that mathematical ideas and play and talk becomes a regular part of their environment, the more they will learn.

Naming Shapes. Knowing the name of a shape is just a small piece of knowing about that shape—like knowing the name of a person is knowing just a small amount about them. It is important that children look carefully at the properties or attributes of various shapes and learn to distinguish between them. Children construct ideas about shapes by manipulating them and using them in play. When introducing *abstract* shapes or drawings of shapes on paper, young children's books and toys tend to start with closed, symmetrical shapes such as equilateral or isosceles triangles, squares, circles, and rectangles. And these materials tend to present these shapes as they would sit on the table, not like  .



Regular shapes with horizontal bases are the shapes most commonly used in children's toys and the ones they are most likely to see or have pointed out to them. With familiar real-world objects, children recognize the outline (shape) even when it is tilted or completely upside down. With abstract shapes, children *may* think that the way it is rotated matters. For example, children who recognize □ as a square may think that ◇ is not a square. It is common, and perfectly fine, to call that second shape a "diamond," but it's important also for children to understand that it is still a square, just in a different orientation. This never happens with physical objects on a table, which children *know* can be dropped down in any position and can be repositioned, but children have no way of knowing whether the name of a shape *drawn on paper* depends on its orientation or not. After all, they will put great effort into learning that p, d and b have different names, even though they are *exactly* the same shape, just in three different orientations. Similarly, they will put great effort into learning that three is written 3, not Ɛ; sometimes orientation matters a great deal. As a result, children may assume that, on paper, □ and ◇ may be different shapes. Children need to learn explicitly that when we ask

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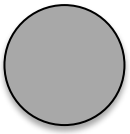
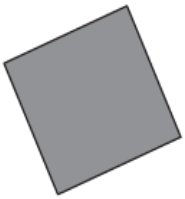
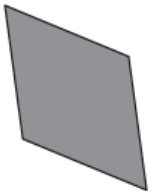
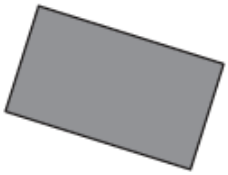
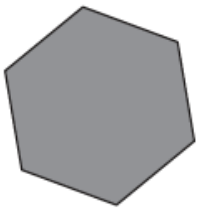
"what *shape* is this?" orientation *doesn't* matter. Otherwise, the confusion that turning a shape will change its name can last into later elementary school. Providing children instruction focused on the shape's defining attributes—such as number and length of sides—helps them create more accurate understandings.

The shapes on the shape cards are intentionally set at various angles to give children experience with shapes whose bases are not horizontally aligned. As children play matching games, *Wild Shapes*, and *Go Fish* with the shape cards, they are practicing identifying and naming the shapes they see. Below are the shapes on the shape cards, their names, and defining attributes.

Shape	Shape Name	Defining Attributes
	Circle	A shape made by drawing a curve that is always the same distance from a center point.
	Square	A shape with 4 straight sides that are all the same length and 4 right angles (like the corners of piece of paper)
	Triangle	A shape with exactly 3 straight sides.
	Rhombus	A shape with 4 straight sides that are all the same length.
	Rectangle	A shape with 4 right angles (like the corners of piece of paper) and 4 straight sides. Opposite sides are the same length and parallel (like train tracks).

	Hexagon	A shape with exactly 6 straight sides.

Shape Attributes. An attribute is broadly defined as a characteristic or quality belonging to a person, thing, or group. When talking to children about attributes it can be helpful to think of using our five senses to 'find' the attributes of things: *What does it look like (eyes)? Sound like (ears)? Smell like (nose)? Taste like (mouth)? Feel like (touch)?* All of our senses give us important information about the world around us. Developing children's ability to identify and define attributes is an important mathematical habit of mind.

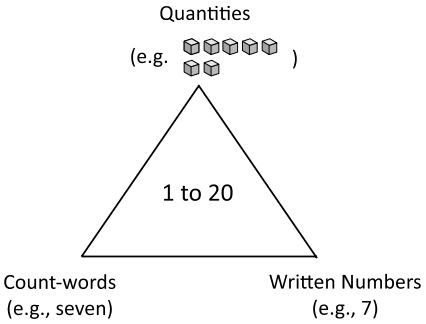
In the shape card games, we are primarily using our eyes to notice the attributes of shapes and identify the same shapes. Two of the decks use color to help children distinguish the shapes. Two of the decks are in black and white to challenge children to identify these abstract objects *just by shape* even when they don't have color to help them. While children naturally notice color (or size) first, those are not a defining attribute of what makes a triangle a triangle or a square a square. To be a square, a shape must have exactly four straight sides, all of equal length, and right-angle corners. (At this age, identifying a right angle as the same shape as the corner of a sheet of paper will have more meaning than naming a number of degrees.) The triangle and quadrilateral Venn diagrams (pages 7 & 8) organize these shapes by their defining attributes and are a helpful reference for you to think about comparing shapes. They are not a reference for the children—children are not expected to see all the relationships among these shapes until middle school or later.

As children are working to name, identify, and match shapes, introduce language about shape as it comes up naturally. Talk about the number and length of sides, the number and size of angles/corners. Since precise angle measure cannot have meaning at this age—children don't know what 90 is, let alone 90 *degrees*—it is enough for them to notice whether angles *match* the corner of a sheet of paper "a right angle," or are "pointier" or "less than a right angle" or "wider" or "more than a right angle." You can even talk about whether the sides are parallel or not using the analogy of train tracks to define parallel lines, or by thinking of the lines "going in the same direction." We want to increase the language and vocabulary children have to clearly and precisely describe shapes (and by extension, the world around them). If they draw letters, we can talk about them with the same language. An H has two parallel lines. An E has *three* parallel lines! The rectangle, square and rhombus often cause confusion, even for adults. Spend a few minutes looking at the attributes of these shapes and helping children distinguish what features the shape *must* have from the features that it *may* have. A rhombus *must* have four equal sides. It *may* have right angles, but it doesn't have to. A square has four equal sides (so it is a rhombus), and it *must* have right angles (so it is a very special rhombus). A rectangle *must* have four sides and four right angles. It *may* have all sides the same length, but the opposite sides must have to be the same length. A square has four sides and four right

angles (so it is a rectangle), and it *must* have all its sides the same length (so it is a very special rectangle).

Recognizing Written Numerals 1-5. These games give children more practice recognizing numerals. Learning to recognize the written numerals is similar to learning to recognize the letters of the alphabet. For letters, they connect the written symbol to the letter name and the sound. For numerals, they connect the written symbol to the number name and the quantity it represents. The figure below shows this relationship. Some children may be ready to start writing numerals themselves just as they are writing some of the letters of their name. You may want to start them by writing the numeral for their age.

In the shape card games, children are identifying and naming the numerals on the cards. Each card also has the corresponding number of shapes on it.



Executive Functions. Executive functions are the cognitive processes that enable children to plan, focus attention, remember information, and juggle multiple tasks. When playing the shape card games, children use three aspects of executive functions—response inhibition, rule shifting (also called cognitive flexibility), and keeping track (an aspect of working memory). These executive functions have been shown to be related to mathematics achievement.

Response inhibition. Response inhibition, sometimes called self-regulation, is the ability to ignore distractions. It also helps children maintain focus on one project for a sustained period of time or over a few days. When playing games, children have to wait to take their turn, inhibiting their own actions while they wait for others. In the shape card games, they have to inhibit looking at shape while they attend to the numeral or inhibit looking at the numeral while they attend to shape. This skill is important in school as children learn to pause, think, and raise their hand before shouting out the answer. Inhibition is particularly important in the early grades as children learn more sophisticated strategies to solve simple problems (such as counting on from 5 when adding 5+3 rather than counting 1,2,3,4,5,6,7,8). Or in later grades, when they are learning to add fractions, children need to stop themselves from just adding all the numbers they see, turning $\frac{1}{2} + \frac{1}{3}$ incorrectly into $\frac{2}{5}$.

Rule shifting (cognitive flexibility). Rule shifting or cognitive flexibility refers to the ability to flexibly shift perspectives or the focus of attention from one attribute of an object to another, or from one rule to another. For example, shifting from focusing on shape to focusing on number when playing *Wild Shapes* is rule shifting. The ability to shift back and forth

between rules and to attend to some aspects of the card while ignoring others is what children are practicing when playing this game.

Working memory. Working memory is the ability to hold information in your head and use that information to solve problems. In *Wild Shapes*, when children are keeping track of the cards in their hand and the card in the discard pile, they are using their working memory. The card in the discard pile changes every time someone takes a turn, so they have to keep updating that information. Likewise, children have to update the information about what cards are in their hand each time they take a turn to pick one up or put a card down. Research has established the importance of working memory as an important predictor of math learning in the early grades. Experience helps build that working memory.

Progression from 3- to 6-Years-Old (End of Kindergarten)

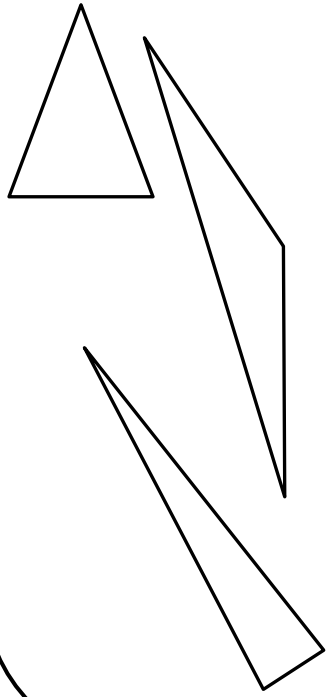
The lines between columns are intentionally fuzzy because the age is approximate. This progression is not to be used as an assessment or checklist, or to judge whether a child is ready to transition to Kindergarten. They represent expectations for children, but each child will reach these indicators at their own pace and their own way. These are meant to help you know what to expect; what learning may come first and what learning may come next for most children.

	@3 years old	@4 years old	@5 years old	@6 years old (end of Kindergarten)
Naming Shapes	Names typical 2D shapes (circles, squares, triangles, rectangles)	Names and describes familiar 2D shapes (circle, square, triangle, rectangle). Begins to recognize less typical examples (i.e. a long skinny triangle is still a triangle because it has 3 sides)	Names and describes most 2D shapes regardless of orientation and size (e.g., familiar shapes plus rhombuses, trapezoids, hexagons)	Correctly names and describes 2D shapes regardless of orientation or size (e.g. familiar shapes plus more such as octagons, parallelograms, convex/concave figures)
Shape Attributes	Notices shapes have sides and corners	Begins to describe shapes by number of sides and corners and sides of same or different length	Describes shapes by specifying number of sides and corners and by indicating whether sides are of same or different length	Compares shapes, describing similarities, differences, and parts using language such as number of sides and corners, and sides of same or different length
Read and write numerals	Identifies numerals as being different than letters and identifies some, such as 3	Reads numerals 1-5	Reads numerals 1-10, begins to write some, such as 1, 3, 7	Reads and writes numerals 0-20 (K.CC.A.3)

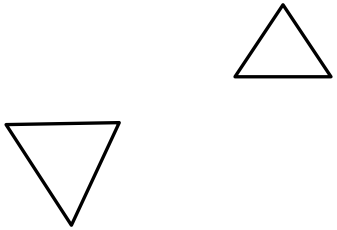
Triangles[1]

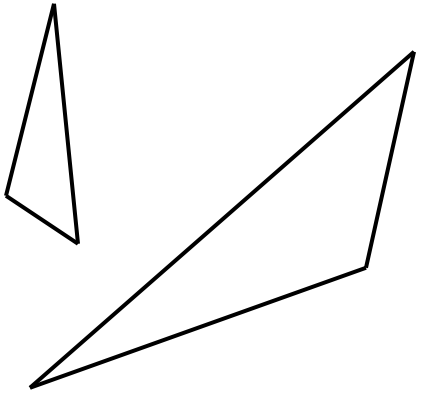
(closed figure with three straight sides)

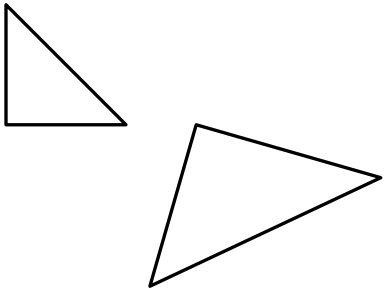
Isosceles
(two sides the same length)



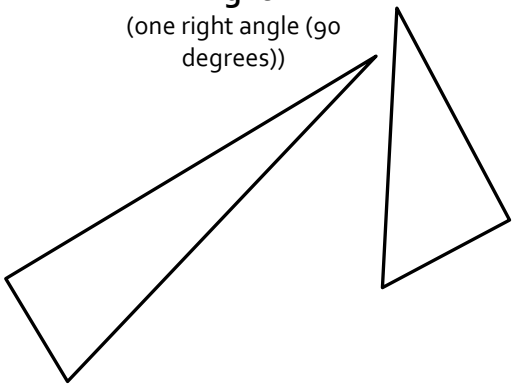
Equilateral
(All three sides the same length and all three angles the same size (60 degrees))





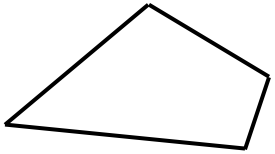


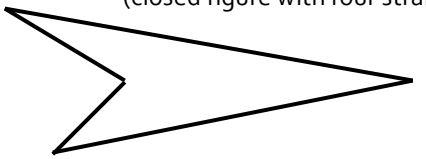
Right
(one right angle (90 degrees))

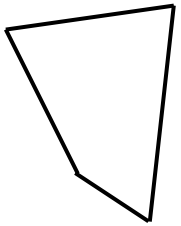


Quadrilaterals

(closed figure with four straight sides)

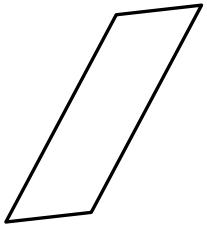


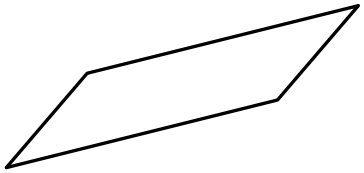




Parallelograms

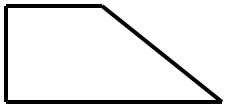
(opposite sides the same length and parallel)





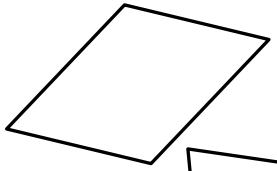
Trapezoids

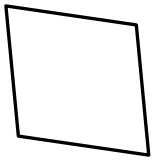
(at least one pair of parallel sides)



Rhombuses

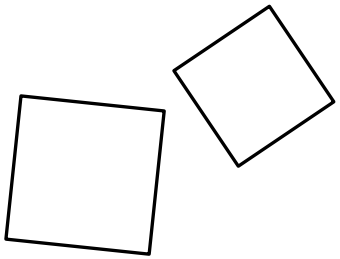
(sides all the same length)





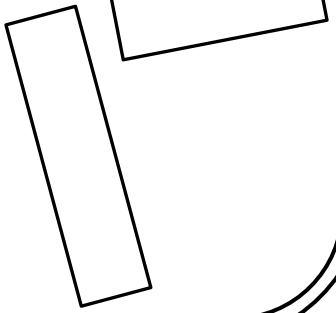
Squares

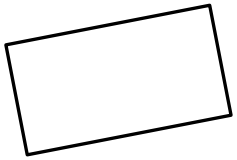
(All four sides the same length and parallel AND four right angles)



Rectangle

(opposite sides the same length and parallel AND four right angles)





Geometric Terms[2]

Term	Meaning
Angle	The shape formed by two lines that meet at a point. That point is called the vertex of the angle; the lines are called the sides of the angle. A right angle matches the corner of a sheet of paper. When we measure in degrees (unimportant at this age), its measure is 90 degrees.
Closed figure	A figure is called "closed" when it separates an inside from an outside and one cannot get from the inside to the outside without crossing a line. A circle is a closed figure. A triangle is a closed figure. The letter U is an open figure: one can get from the "inside" to the "outside" without crossing the line.
Mirror Symmetry	A shape has mirror symmetry if the shape can be folded in such a way that the two parts lie precisely on top of each other, matching exactly. A rectangle can be folded side-to-side, splitting the other two sides in half, and the two parts will match exactly. That fold is called the *line of symmetry*. If you fold a rectangle (that isn't a square) along its diagonal, the two parts are the same size and shape, but don't lie perfectly on each other matching exactly. That fold line is *not* a line of symmetry, but because the rectangle has *other* lines of symmetry, we say it has mirror symmetry.
Orientation	How a figure is turned compared to a reference line.
Parallel	Lines have the same orientation and remain the same distance apart (like railroad tracks).
Polygon	A closed 2D figure with three or more straight sides.
Rotational symmetry	A figure has rotational symmetry if it can be turned less than a full turn and fit on itself exactly. A square, for example, can be rotated one fourth of the way around (90°) and fit on itself exactly.
Concave	At least one corner looks caved in; an interior angle of the polygon is greater than 180 degrees.  and  are six-sided concave polygons.
Convex	Every interior angle is less than 180 degrees; no corner pokes into the figure.  is a five-sided convex polygon.

Polygon Names

Triangle	3 straight sides
Quadrilateral	4 straight sides
Pentagon	5 straight sides
Hexagon	6 straight sides
Septagon	7 straight sides
Octagon	8 straight sides

[2] Children may find these terms interesting but there is no expectation that they learn them.

OBSERVATIONS TO MAKE WHILE PLAYING

As you observe what your children are doing, support them to take the next step in their mathematical thinking by modeling, questioning, and explaining.

Do children		*Do children*
• Focus on number of sides and corners when identifying a shape?	OR	• Focus on how similar a shape looks to a familiar shape?
• Recognize shapes even if the position is "upside down" or unfamiliar?	OR	• Correctly identify only shapes that are in a more "typical" position?
• Match shapes easily?	OR	• Need assistance to match shapes?
• Math numbers easily?	OR	• Need assistance to match numbers?
• Able to switch between matching number and shape?	OR	• Have trouble switching between the match by number and match by shape rules?
• Make strategic decisions when taking their next move games like in Wild Shapes or Go Fish?	OR	• Pick the first card they notice when making a move in games like Wild Shapes or Go Fish?

BOOK LINKS

Pete the Cat and His Four Groovy Buttons *by Eric Litwin and James Dean*
Pete the Cat is fun and loveable. At 8am, Pete puts on his favorite shirt with 4 groovy buttons. His buttons inspire him to sing, "My button, my buttons, my **four** groovy buttons, my buttons, my buttons, my **four** groovy buttons." But as he goes about his day, one-by-one his buttons pop off his shirt. Does Pete cry? No, he keeps going and keeps singing his song. This book has lot of fun math in it as Pete counts down his buttons from 4 to 0. The song he sings repeats in a predictable way (a pattern) as the number of buttons goes down by one each time. The illustrator includes the subtraction equation (such as, 4 – 1 = 3) as each button pops off. You can show the children the equation and explain that they will learn to write mathematical equations like that in kindergarten.

Children can draw their own shirt pictures and glue dots on them for buttons. Then they can write on them the number of buttons they have. For example, "I have 5 buttons on my blue shirt. If I lose one, I'll have 4!"

The Greedy Triangle by Marilyn Burns
In this book, a triangle explores all the different things he can make as a triangle—a sailboat, a roof, a piece of pie. But he decides he is bored with being a triangle and goes to the shape shifter to get another side and become a quadrilateral! He explores all the things he can be as a square—a computer screen, a picture frame, a game square. Again, he gets bored and goes to the shape shifter to get another side. This continues until the shape is almost round and rolls away from all his friends. In the end, he decides he wants to go back to being himself, a triangle. When reading this book, have fun with children exploring all the different shapes but don't worry about them remembering the names for all these shapes. Just use the terms naturally as they arise and the children will pick up some of the words the same way they build the rest of their vocabulary.

Art extensions. Have children focus on one shape with different number of sides at the art table. You can put out straight edges (such as rulers) and tracing shapes to help them get started. One day have them focus on triangles—making 3-sided shapes of all different sizes and length of sides. They can create sailboats, buildings, ladders, or anything they can imagine using triangles. Another day, have children focus on quadrilaterals making windows, buildings, boxes, books, toys, or anything they can imagine. Continue with pentagons, 5 sided shapes.

Shapes (Math Counts) by Henry Pluckrose
A great book to get children thinking about the shapes they see in the world around them. The first few pages ask children to run their finger around the edge of a square, circle, rectangle, hexagon, and triangle. Then there is a page showing three different squares and the next page showing five different triangles that ask children to look at how they are similar and different. The next part of the book is great for talking about going on a shape hunt---there are

photographs where children can find rectangles, squares, triangles, circles in the real world—even hexagons in the honeycomb. On page 16, the author introduces the word *tessellation* to describe shapes that fit together without leaving spaces. While the word is probably new to children, its probably a concept they have experienced when building in the block area or making designs with the pattern blocks.

Math extension. Go on a shape hunt in your classroom, your school, and outside. Find circles such as clocks, knobs, or stools. Find rectangles in windows, art paper, or photographs. Have children cut out their own shapes and glue them onto paper to make pictures and designs.

Home activity. Ask children and caregivers to go on a shape hunt at home.

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