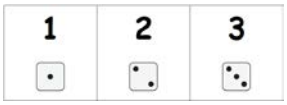


ABOUT THE MATH

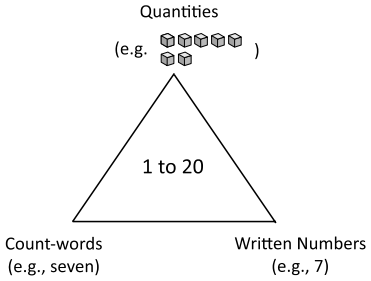
If you watch and listen to how students interact with the games, you can learn a lot about what they know and what they're ready to learn. Once you see what they can do, you can help them take the next step. In this game, children practice:

- Recognizing written numerals 1-6 (possibly up to 12)
- Placing numeral cards in the correct order
- Using one-to-one correspondence when counting
- Identifying the number of dots without counting (subitizing)
- Composing and decomposing numbers: Identifying the new number created when numbers are combined or separated

This section discusses some of the mathematical skills that children are building as they play. In mathematics, just as in their language and social/emotional skills, preschool children vary greatly in what they know and are able to do depending on their development and the experiences they've had. The more that mathematical ideas and play and talk becomes a regular part of their environment, the more they will learn.

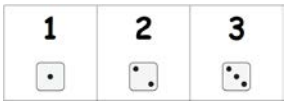
Recognizing written numerals 1-5 (possibly up to 12). Learning the written numerals is similar to learning the letters of the alphabet. Children need repeated exposure and need to connect the written symbol to its meaning. For letters, they connect the written symbol to the letter name and the sound. For numerals, they connect the written symbol to the number name and the quantity it represents. The figure below shows this relationship. Some children may be ready to start writing numerals themselves just as they are writing some of the letters of their name. You may want to start them by writing the numeral for their age.

Pointing to written numerals while counting reinforces the link between the symbol and its name. The game *Two Numbers* is designed to help children connect the symbol to the quantity it represents and to the number name. As children play, they count the dots on the dot cubes and say the number names. They then identify the number name they said with the written numerals and the number of dots on the card(s) they can turn over.



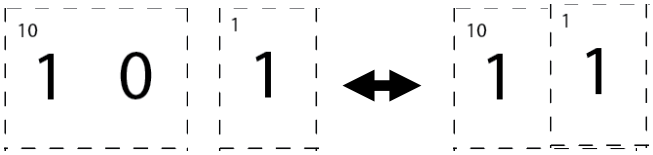
Talking about 2-digit numbers (i.e. "What does the "1" in the number 12 mean?").
When we talk to children about two-digit numbers, we need to be careful about the language that we use. The 1 in 12 stands for 10; the written number 12 stands for 10 + 2. When talking to

This work has been supported by funding from the National Science Foundation and Heising-Simons Foundation.
Citation: Reed, K. E., & Young, J. M. (2017). *Games for Young Mathematicians: About Two Numbers Games.* Waltham, MA: Education Development Center, Inc. http://ym.edc.org/



young children about two-digit numbers, we can say: "12, 10 and 2 make 12. The 1 in 12 stands for 10". Sometimes we might say: "1 and 2 make 12." When we are talking about writing 12, then yes, the digits 1 and 2 next to each other are 12. But if we are talking mathematically, 1 and 2 doesn't make 12, it makes 3. The language is not precise so it can be confusing for young children. You may want to use your fingers to show them how 1 plus 2 is 3 and compare that to how 10 and 2 make 12 (with your fingers). Or you can take 12 objects and group them into 10 and 2.

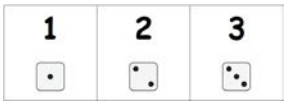
Preschool is not the time for children to formally learn about place value. In preschool, we just want to give children experiences with putting together and taking apart small numbers. When children are in first grade, these preschool experiences will help them learn that a group of tens ones can be grouped and counted as a single unit (one ten). In first grade, they learn that ten ones can be grouped and counted as a single unit (one ten). They may also see layered place value cards such as the ones below. But this formal instruction is for first grade, not for preschool.



Reciting number words in the correct order. Some children will know the number words but not be able to recite them in the correct order (i.e. 1, 2, 6, 4, 5, 10) or routinely skip a certain number (i.e. 1, 2, 3, 5, 6—always skipping 4). Recite the number list frequently in the day (when waiting for the bathroom, passing out or cleaning up materials) and listen to children recite it, helping them where needed. Many children benefit from hearing rhythm to the list: "one, two, three; four, five, six;…" Tone can give salience too, particularly when children are stuck on a certain number. Saying: "Let's count together. One, two, three, four" each as if they're anticipating something special, and then "*five!*" at the end as if it were the special surprise we were all waiting for. Then: "See, *five* comes after four! Now you do it!"

The *Two Numbers* game gives children enjoyable practice reciting the number list while pointing to cards that have both the numeral and the matching number of dots. By ordering the cards from 1 to 5, they are rehearsing the correct number order and attaching a visual image to the words they are saying.

One-to-one correspondence. When children have one-to-one correspondence that means they connect exactly one counting word to exactly one object. For example, a child counts three cubes as "one, two, three," touching each object only once and assigning only one counting word to each. This is a more complicated endeavor than it appears, because it requires two kinds of matches: (1) "matching a moment of time when the action occurs and a word is said"; and (2) "matching in space where the counting action points to an object once and only once" (NCTM, 2010, p. 13). Young children often recite the words and touch the objects at different rates, going through the right actions—reciting and touching—but not yet with the right meaning. You can model the correct counting to help them remember that each



object needs one point and one number word; you don't skip any. Or you can say, "You might have missed one. Can you check?"

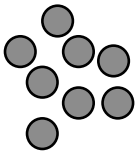
Sometimes children seem to think that a line of five blocks with big spaces between them has "more" than another line with the same number of blocks but more tightly spaced. They may be confusing the size of the configuration of objects and the total number of objects. You can try having them imagine that the objects are small toys or treats (or something else they'd want a lot of), and ask which collection they would want and why. Without negating what the child *has* done correctly—counting the number and assessing the visual "size"—this gives the child one extra experience thinking about quantity; experience, over time, will solidify the child's logic.

The *Two Numbers* game gives children an opportunity to count the dots on the cards and on the dot cubes (dice). While they are playing, notice if they 'just know' how many dots are on the cards and cubes (subitize), are counting each dot with one-to-one correspondence, or if they are counting inaccurately.

Cardinality. Children may count accurately (i.e., with one-to-one correspondence) but not yet recognize that the last number they said tells them the total amount in the collection. They have not yet achieved what is called 'cardinality.' After children count a set, we can ask them "How many do you have?" If they state the total number, they are showing an understanding of cardinality. If they recount the set, then they may not yet understand cardinality. Of course, they may also just have forgotten, or they may interpret the question to mean that they got the number wrong, and should check. They may even understand "How many" not as a call to answer, but as a call to *act*, to *count*. We can help them by restating the total number after counting: "1, 2, 3. We have 3 pennies!"

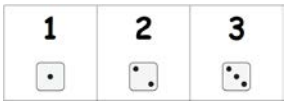
Two Numbers is another opportunity for children to practice cardinality. They need to figure out "how many?" on the dot cubes every roll.

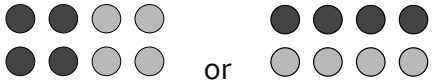
Subitizing. Subitizing is *instantly seeing how many* for small quantities. For example, even preschoolers, for the most part, instantly recognize the number of fingers here  without needing to count "one, two" to know how many. But there are limits to what we see automatically. Even practiced adults rarely subitize quantities greater than 4 or 5. For example, this collection of dots is hard to subitize. We have to count, or mentally chunk it in parts, to know how many.



As children gain experience, they begin to see clusters they can subitize within a larger quantity (up to about 10) and can use those chunks to recognize the quantity immediately. For example, while 8 dots is generally too large to see immediately as 8, if they are arranged in a way that lets us easily perceive them as two groups of four, and if we 'know' that two 4s make 8, we "instantly" recognize the 8.

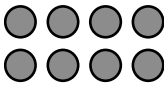
Reed, K. E., & Young, J. M. (2017). *Games for Young Mathematicians: About Two Numbers Games.* Waltham, MA: Education Development Center, Inc. http://ym.edc.org/





We *perceptually* subitize the two groups of four—two and four are both within perceptual limits—and then we *conceptually* subitize the 8.

Even without color to help us group the objects, a well-arranged collection can let our minds do that grouping for us, letting us quickly see how many.



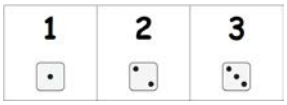
People can learn, in a limited way, learn to clump objects visually even when they are not arranged conveniently. For example, look back at that first mess of dots and see if you can see it in two groups you instantly know the size of. This is learnable, but more like a job skill (perhaps useful for quickly taking attendance) and not something worth spending children's time on.

You can incorporate practice with subitizing throughout the day. For example at snack time, "How many crackers do you have?" or at center time "How many dinosaurs/blocks/cars/dolls do you have?"

The game *Two Numbers*, encourages children to quickly 'see' how many dots there are on the number cubes—and even to add the dots on the two cubes together if they are ready for that.

Comparing Numbers. Children must have experiences that help them develop a 'mental number line'—a spatial representation of quantity that helps them reason about which is more and how much more. Children can reason that 7 is more than 5 because 7 comes later in the counting sequence and on the number line. Later, they can determine how much more 7 is than 5, either by picturing its "distance" from 5 on the number line, or by knowing that 5 is contained within 7 (a part of 7) and the "other part," the leftover, is 2. Studies have found that lower income children can be significantly behind their middle-income peers in their knowledge of counting, comparing numbers, and their development of a mental number line (Jordan, Kaplan, Olah, & Locuniak, 2006; Starkey, Klein, & Wakeley, 2004). Some research has explored whether this could be from less experience playing board games that involve movement from space to space in a number-line-like way; and/or less experience with conversations involving number, discussing how many, or how many more or less?

Most people tend to exaggerate the distances between smaller, more familiar numbers—1, 2, 3, 4—and underestimate differences between numbers at the higher end. Think about how "far apart" one hundred and one thousand are. Are one thousand and one million similarly spaced? One million and one billion? And this is, in fact, built into us as human beings as a practical way of understanding difference. The difference between 1 and 2 *is* more



significant (to us) than the difference between, say, 96 and 98. In a collection as large as 98, you'd hardly miss 2 if you lost them. But losing 1 of 2 things is losing a lot! There's a huge difference between a 7-year-old and her 4-year-old sister, but by the time they are 57 and 54, they are "the same age." But computation involves thinking about the precision as well, and helping children gain an understanding of the number line and relationships between numbers on the number line is important for later mathematical reasoning and arithmetic. In preschool children are working with small quantitates and later that understanding will extended to large numbers.

In *Two Numbers*, children are building their understanding that bigger numbers (like 5) are composed of smaller numbers (like 2 and 3); the larger quantities have more dots. They can see this on the cards and will see it when they roll the dot cubes. They can also see that each counting number is just one more than the previous number.

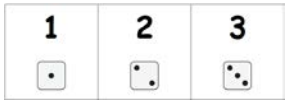
Composing and Decomposing Numbers. Through everyday experiences, young children learn that a whole is made up of parts and that the parts are smaller than the whole. (There can be 4 paintbrushes and if 3 of them are in use, only 1 paintbrush is left). Children also learn early that objects can be combined in different orders to produce the same result—you can put the 3 blue trucks or the 3 red trucks in the bucket first but you still get 6 trucks. (That is what they will later learn as the commutative property.) By 4 or 5 years of age children can recognize that smaller numbers are 'hiding inside' larger numbers, seeing, for example, 2 and 3 hiding inside 5 (as are 4 and 1).

The game *Two Numbers* is helping children build on these early understandings of addition. Although many children will not be ready to add the dots across the two number cubes, some children will choose to count the dots on the two dot cubes together and turn over the card that represents that amount. This is early practice with addition. If children are ready you can tell them they can take away the number of dots on one cube from the number of dots on the other cube, getting one kind of experience with subtraction. When they are thinking explicitly about taking dots away (or covering them up), they can clearly only do that by "taking" the smaller number from the larger. You can tell them that when they are older (somewhere between 4th and 6th grade) they will learn ways to think about subtracting larger numbers from smaller ones—you get negative numbers!

Reed, K. E., & Young, J. M. (2017). *Games for Young Mathematicians: About Two Numbers Games.* Waltham, MA: Education Development Center, Inc. http://ym.edc.org/

Games for Young Mathematicians

Two Numbers

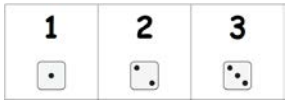


Progression of Number Concepts from 3- to End of Kindergarten

The lines between columns are intentionally fuzzy because the age is approximate. This progression is not to be used as an assessment or checklist, or to judge whether a child is ready to transition to Kindergarten. The information here shows a common (not universal) trajectory of learning, and rough ages at which children reach each point in the trajectory. Children will reach these indicators at their own pace and their own way. These are meant to help you know what to expect; what learning may come first and what learning may come next for most children.

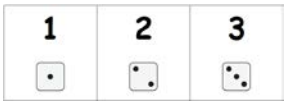
	@3 years old	@4 years old	@5 years old	End of Kindergarten Common Core Standard
Verbally count	Recites number names to 10 with occasional errors	Recites number words to 20 with some errors especially in the teens	Recites number words to 40 with some errors especially in the teens	Counts to 100 by ones and tens (K.CC.A.1)
Count objects	Begins to use one-to-one correspondence for small groups of objects (under 5)	Uses one-to-one correspondence when counting (up to 10 objects)	Uses one-to-one correspondence when counting	Uses one-to-one correspondence when counting (K.CC.B.4.A)
Cardinality	Begins to understand that the last number tells the number of objects in a group	Understands that the last number name used tells the number of objects counted up to 6 things.	Understands the last number name used tells the number of objects counted. Can count out n objects up to 10.	Counts to answer how many for up to 20 objects arranged in a line, array, circle, or up to 10 in a scattered configuration. Can count out n objects up to 20. (K.CC.B.5)
Subitizing	Begins to recognize the number of objects in a group of two or three without counting (subitizing)	Quickly sees how many for 1, 2, and 3 objects (subitize). May begin to subitize visually or conceptually up to 5 objects (by seeing 2 and 3)	Begins to see how many with 1-10 objects when they are in a familiar arrangement; begins to use chunking for numbers 6-10 with a 5 group (array, fingers, dice pattern)	Quickly sees how many with 1-10 objects when they are in a familiar arrangement; uses chunking for numbers 6-10 with a 5 group (array, fingers, dice pattern)
Read and write numerals	Identifies numerals as being different than	Reads numerals 1-5	Reads numerals 1-10, begins to write some, such as 1, 3, 7	Reads and writes numerals 0-20 (K.CC.A.3)

Reed, K. E., & Young, J. M. (2017). *Games for Young Mathematicians: About Two Numbers Games.* Waltham, MA: Education Development Center, Inc. http://ym.edc.org/



	letters and identifies some, such as 3			
Compare numbers	Uses language to compare the number of objects in two groups (more, less, same)	Begins using strategies to find which is more for two numbers ≤ 5	Uses counting to find which is more for two numbers ≤ 5. Uses the words less (fewer) than/more than/same as	Identifies whether the number of objects in one group is greater than, less than, or equal another group of objects. Compares two written numerals between 1 and 10 (K.CC.C.6 & 7)
Composing and Decompos-ing Numbers	Knows the whole is bigger than the parts, but may not correctly quantify	Beginning to know number combinations up to 4 or 5 (3 has 2 and 1 in it)	Uses objects or fingers to decompose small numbers (3, 4, 5) into its parts (5 has 4 and 1 and 3 and 2 inside it). Names parts of numbers up to 5	Decomposes numbers to 10 into pairs using objects, drawings, and/or equations. Knows the pairs that make 10. Fluently adds and subtracts within 5. (K.OA.A.2 & 5)
Counting on				Counts on from a given number instead of starting at 1 (e.g., starts at 3, counts 4, 5, 6) (K.CC.A.1)

Reed, K. E., & Young, J. M. (2017). *Games for Young Mathematicians: About Two Numbers Games.* Waltham, MA: Education Development Center, Inc. http://ym.edc.org/

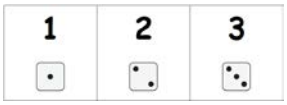


OBSERVATIONS TO MAKE WHILE PLAYING

As you observe what your children are doing, support them to take the next step in their mathematical thinking by modeling, questioning, and explaining.

Do children		*Do children*
• Put the cards in the correct order?	OR	• Make mistakes putting cards in order?
• Count dots one by one accurately?	OR	• Make mistakes in their counting such as skipping or double-counting dots?
• Arrive quickly and efficiently at how many (subitize)?	OR	• Count one-by-one?
• Recognize the numerals 1 to 5? To 10? To 12? (Which ones?)	OR	• Not yet recognize the numerals?
• Consistently recognize the dot patterns on the dice?	OR	• Not yet retain the dot patterns on the dice?
• Recognize the number of dots on one cube and continue that count while pointing to the dots on the other cube?	OR	• Count all of the dots starting at one?
• Know some totals without counting?	OR	• Rely on counting to find the total?

Reed, K. E., & Young, J. M. (2017). *Games for Young Mathematicians: About Two Numbers Games.* Waltham, MA: Education Development Center, Inc. http://ym.edc.org/



BOOK LINKS

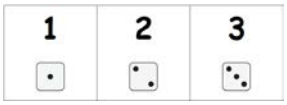
Mouse Counts *by Ellen Stoll Walsh*
The snake is gathering up the mice and counting them ("1, 2, 3", up to 10) as he puts them in a jar to eat later. When he goes off to gather up one really big mouse, the 10 mice tip over the jar and escape! They "uncount" themselves 10, 9, 8, 7, 6, 5, 4, 3, 2, 1. This adorable story is great for practicing counting forwards and backwards.

To extend the learning in this book you could make 10 mice. Have children act out the counting in the book by placing them in a clear container, then have them escape and counting backwards.

How Many Snails: A counting book by Paul Giganti, Jr., illustrated by Donald Crews
Walking to the meadow, lake, library, park, bakery, toy store, and other stops, the author wonders 'how many?' about a variety of different objects and in different combinations; such as: "How many snails were there? How many snails had striped shells? How many snails had striped shells and stuck their heads out?" This is a fun counting book to use as a read aloud and then for children to browse on their own counting all the objects and sorting them into different groups. It specifically targets the skills of cardinality—knowing how many in all in a group. This skill is associated with stronger mathematics achievement later in school, so make sure to practice it in preschool!

Math Counts: Numbers *by Henry Pluckrose*
This book is great for using a few pages at a time to talk about all the places we see and use numbers in our world. You can have the children go on a Number Hunt to find and identify the numbers on the page and then find and identify numbers in the classroom, at the center, outside, and at home. Children could create pictures that include numbers they like.

Reed, K. E., & Young, J. M. (2017). *Games for Young Mathematicians: About Two Numbers Games.* Waltham, MA: Education Development Center, Inc. http://ym.edc.org/



REFERENCES

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics (CCSSM). Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Griffin, S., Case, R., & Capodilupo, A. (1995) Teaching for understanding: The importance of the Central Conceptual Structures in the elementary mathematics curriculum. In A. McKeough, J. Lupart, & A. Marini (Eds.), *Teaching for transfer: Fostering generalization in learning* (pp. 121-151). Mahwah, NJ: Lawrence Erlbaum Associates.

Jordan, N. C., Kaplan, D., Nabors Oláh, L., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child development*, *77*(1), 153-175.

National Council of Teachers of Mathematics (NCTM). (2010) *Focus in Prekindergarten: Teaching with curriculum focal points*. Reston, VA: Author.

Office of Head Start (2015). *Head Start Early Learning Outcomes Framework: Ages Birth to Five*. Arlington, VA: Administration for Children and Families, U.S. Department of Health and Human Services. Retrieved from http://eclkc.ohs.acf.hhs.gov/hslc/hs/sr/approach/pdf/ohs-framework.pdf

Rhode Island Department of Education (2013, May). *Rhode Island Early Learning and Development Standards.* Retrieved from http://rields.com/wp-content/uploads/2014/08/RIELDS_standards_web.pdf

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, *19*(1), 99-120.